Information Design Perspective on Calibration

Part II – Applications

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Joint tutorial with Yiding Feng, HKUST

Recap

Definition [Dawid, JASA'82][Foster Vohra, Biometrika'98]**.**

Predictor F is **calibrated** if for **every** prediction $q \in [0,1]$

$$\mathbb{E}[Y|F = q] = q$$

Calibrated predictor can be viewed as an signaling scheme

- **Data distribution** $D \in \Delta(\mathcal{X} \times \{0,1\})$
  - State Space: $\{p_X\}_{X \in \mathcal{X}}$ where $p_X = \mathbb{P}(Y = 1 \mid x = X)$
  - Prior: $\{\mathbb{P}_{x \sim D}(x = X)\}_{X \in \mathcal{X}}$
- **Predictor:** $F: \mathcal{X} \mapsto \Delta([0,1])$ (or equivalently $\{p_X\}_{X \in \mathcal{X}} \to \Delta([0,1])$) is **a signaling scheme over posterior means**
  - The prediction q is a signal
  - Calibration requires that $q =$ induced **posterior mean**

Applications

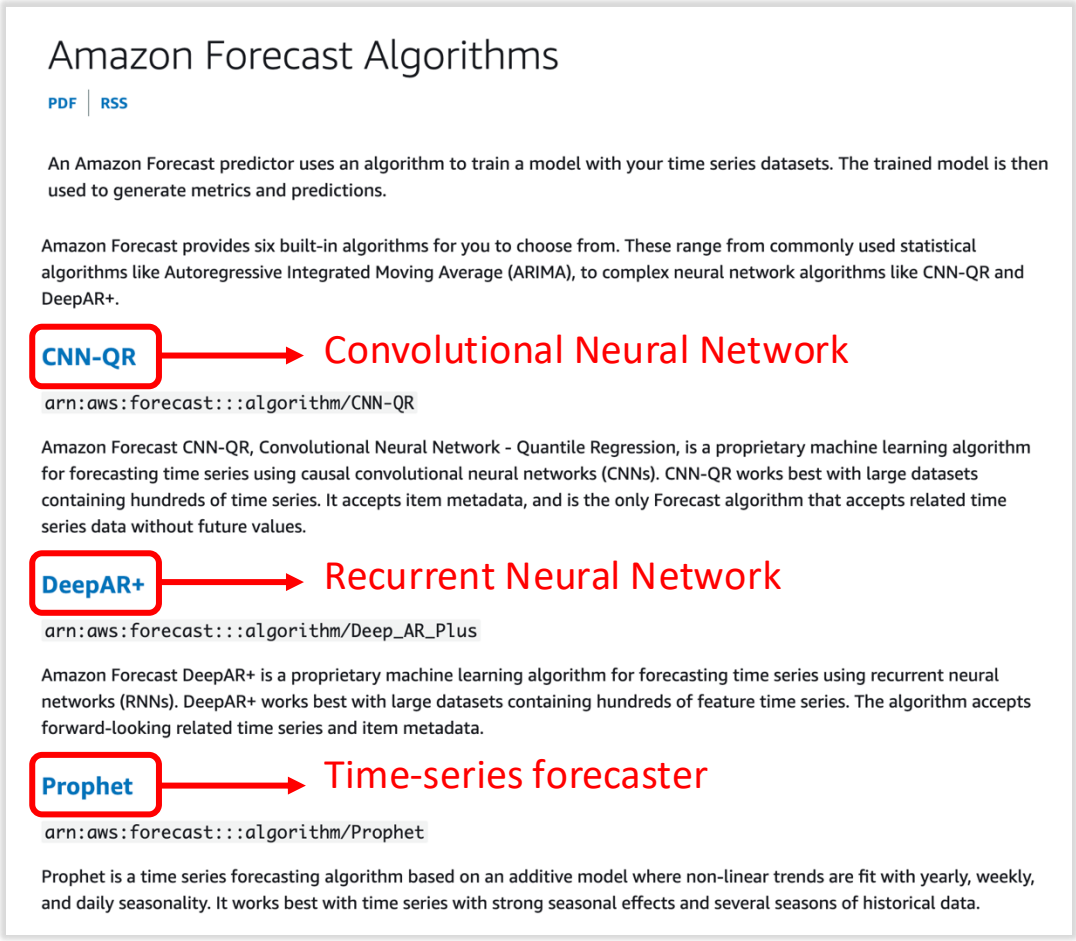
- InfoGap to compare different predictors

- Calibrated signaling in digital auctions

Key technical component: formulating the optimization problem as an optimal transportation problem.

Motivation: Comparison between Multiple Predictors

In reality, platform choose between **multiple predictors** **for their downstream users**



Motivation: Comparison between Multiple Predictors

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Show 10 entries Search:

Rank	Org	Model	Overall
1		Superforecaster median forecast	0.093
2		Public median forecast	0.129
3		Gemini-2.5-Flash-Preview-04-17 (zero shot)	0.130
4		O3-2025-04-16 (zero shot)	0.135
5		GPT-4.5-Preview-2025-02-27 (zero shot)	0.136
5		Claude-3-7-Sonnet-20250219 (scratchpad)	0.136
5		O3-2025-04-16 (scratchpad)	0.136
8		GPT-4.5-Preview-2025-02-27 (scratchpad)	0.140
9		DeepSeek-R1 (scratchpad)	0.143
9		Grok-beta (zero shot)	0.143

Showing 1 to 10 of 83 entries
last updated 2025-12-07 Previous Next

ForecastBench

Market Return

Average Return measures the decision value of a probabilistic prediction by simulating the expected profit of an optimal betting strategy based on the prediction, under the market conditions at the time of prediction and a specified level of risk aversion.

Rank	Model	Provider	Events	Average Return	Confidence Interval (90% CI)
1	GPT-4o	OpenAI	2,011	**90.87%**	±0.1240
2	o3	OpenAI	2,069	**90.30%**	±0.1310
3	GPT-5 (high)	OpenAI	2,037	**88.19%**	±0.0500
4	Qwen 3 235B	Qwen	2,051	**87.67%**	±0.1300
5	GPT-4.1	OpenAI	2,008	**86.95%**	±0.0467
6	Claude Sonnet 4 (Thinking)	Anthropic	2,102	**86.92%**	±0.1301

Prophet Arena

Desiderata for Comparisons

- A decision-making task is specified by $(\mathcal{A}, u)$
  - $\mathcal{A}$ is action set,
  - Decision maker's utility function: $u: \mathcal{A} \times \{0, 1\} \to \mathbb{R}$

- Can we say a predictor is **always** useful than another predictor?
  - DM's Payoff from **acting by trusting** a predictor F: $U(F) = \mathbb{E}_{Y,q\sim F}[u(a^*(q), Y)]$
  - Ordinal comparisons (Partial order)

- Can we bound, to what extent, how much a predictor is **worse** than another predictor?
  - Cardinal comparison

Example I

Suppose $Y \sim \mathrm{Bern}(0.5)$.

Predictors	ECE	"Usefulness for decision making"
 Predict mean $\mathbb{E}[Y]$	0	
 Predict 1 iff $Y = 1$	0	
 Predict 1, 0 uniformly	0.5	
 Predict 0 iff $Y = 1$	1	

Example I

Takeaways

Even with **same ECE**, one predictor may **dominate** another predictor for decision making.

Example II:

Suppose $Y \sim \mathrm{Bern}(0.5)$.

Predictors	ECE	"Usefulness for decision making"
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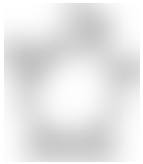 Predict 1 w.p. 0.99 if $Y = 1$ Predict 0 w.p. 0.99 if $Y = \mathbf{0}$ Predict $\mathbb{E}[Y] - 0.001$ otherwise	> 0	

Example II:

Takeaways

One **miscalibrated** predictor may **dominate** another calibrated predictor for decision making.

[Questions]

Can we **compare** any two (possibly miscalibrated) predictors based on how "useful" they are to the **decision-making problems**?

Our Main Results

Our Results [Informal]. We provide a measure, referred to as ***informativeness gap*** between **any** two (**possibly miscalibrated**) predictors, that allows both **ordinal/cardinal comparisons**

Informativeness Gap

Definition. [Feng, Qian & Tang, EC'26]

Given two predictors F and G, **informativeness gap** $\mathrm{INFOGAP}[F, G]$ of G relative to F is

$$\mathrm{INFOGAP}[F, G] \stackrel{\text{def}}{=} \sup_{u \in \mathcal{U}} U(F) - U(G)$$

where

- $\mathcal{U}$: all decision tasks with bounded utility differences
- $U(F)$: expected payoff by naively best responding to prediction $p \sim F$

$$U(F) = \mathbb{E}_{Y, q \sim F}[u(a^*(q), Y)]$$

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- In general, $\mathrm{INFOGAP}[F, G] \neq \mathrm{INFOGAP}[G, F]$
- If $\mathrm{INFOGAP}[F, G] \to 0 \Longrightarrow$ predictor G is more **useful** than F, or **more informative**

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- If $\mathrm{INFOGAP}[F, G] \to 0 \implies$ predictor G is more **useful** than F, or **more informative**
- $\mathrm{UCal}[G] = \mathrm{INFOGAP}[\delta_{(\lambda)}, G]$ where $\lambda = \mathbb{E}_{p \sim G}[p]$ Kleinberg, Leme, Schneider, Teng COLT'23
- $\mathrm{CDL}[G] = \mathrm{INFOGAP}[G^{\mathrm{Bayes}}, G]$ where G^{Bayes} is the corresponding true distribution induced by G Hu & Wu, FOCS'24

Blackwell's Informativeness

Blackwell's Informativeness: When F, G are **calibrated**, G **Blackwell dominants** F if and only if $U(G) \geq U(F)$ for all decision task u

[Blackwell, Annals of Mathematical Statistics'53]

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[Blackwell, Annals of Mathematical Statistics'53]

- When F, G are **calibrated**, $\mathrm{INFOGAP}[F, G] = 0 \Leftrightarrow G$ **Blackwell dominants** F
  - Blackwell order: **ordinal** comparison (**partial order**) over **calibrated** predictors
- InfoGap: **cardinal** comparison over **any two possibly miscalibrated** predictors

Dual Characterization of Infor Gap

- Recall classic earth mover's distance (aka., Wasserstein distance)

$$\mathrm{EMD}[f,g] \stackrel{\text{def}}{=} \inf_{\pi \in \Pi(f,g)} \int_0^1 \int_0^1 \pi(p,q) \cdot |p-q| \cdot \mathrm{d}q \cdot \mathrm{d}p$$

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- Recall classic earth mover's distance (aka., Wasserstein distance)

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Theorem I [Feng, Qian & Tang, EC'26] Given two **calibrated** predictors F and G, $\mathrm{INFOGAP}[F,G]$ equals to corresponding **relaxed earth mover's distance**:

$$\mathrm{INFOGAP}[F,G] = \mathrm{REMD}[f,g] \stackrel{\text{def}}{=} \inf_{\pi\in\Pi(f,g)} \int_0^1 \left| \int_0^1 \pi(p,q)\cdot(p-q)\mathrm{d}q \right| \mathrm{d}p$$

where

- f, g: prediction distribution PDF of predictors F, G
- $\Pi(f,g)$: all couplings (matching marginal) for distributions f, g

- Thus, $\mathrm{REMD}[f,g] \le \mathrm{EMD}[f,g]$

Closed-Form for REMD

Theorem II. [Feng, Qian & Tang, EC'26]

Given two **calibrated** distribution f, g with [0,1] support and identical means,

$$\mathrm{REMD}[f,g] = 2 \cdot \max_{t\in[0,1]} S_f(t) - S_g(t)$$

where

- $S_f(t)$ is super-cumulative distribution function (SCDF) defined as

$$S_f(t) \stackrel{\text{def}}{=} \int_0^t \int_0^s f(z) \cdot \mathrm{d}z \cdot \mathrm{d}s$$

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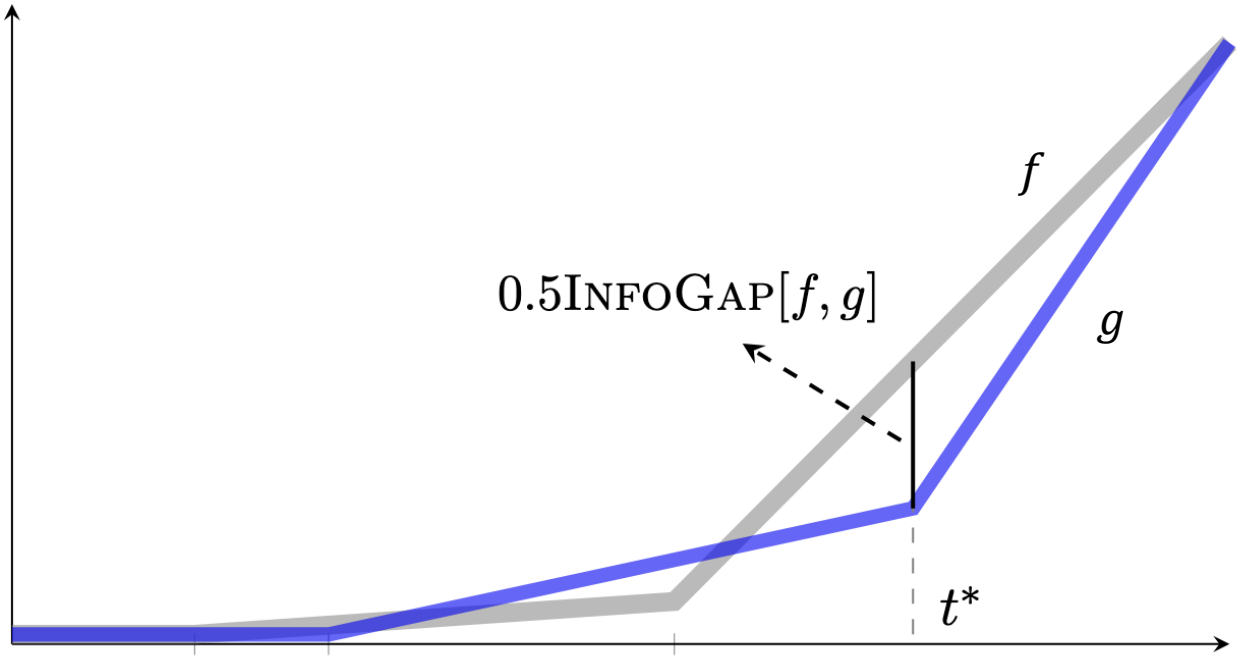
- Implication [Blackwell'53]: g Blackwell dominates f iff $S_g(t) \geq S_f(t)$ for all t
- Implication: $\mathrm{REMD}[f,g]$ (and $\mathrm{INFOGAP}[F,G]$) admits **polynomial** (in $1/\epsilon$) **time complexity** and **sample complexity**

A graphic illustration of $\mathrm{REMD}[f, g]$



$$\mathrm{REMD}[f, g] = 2 \cdot \max_{t \in [0,1]} S_f(t) - S_g(t)$$

- We also generalize **Theorem I** and **Theorem II** when F, G are possibly miscalibrated

Characterization of Infor Gap

Theorem III [Feng, Qian & Tang, EC'26] Given two **miscalibrated** predictors F and G,

$$\text{INFOGAP}[F,G] = \text{REMD}^{\text{MisC}}[f,g,\kappa_f,\kappa_g] \stackrel{\text{def}}{=}$$

$$\inf_{\pi\in\overline{\Pi}(f,g)} \int_0^1 \left| \int_0^1 \pi(p,q)\cdot(p-q)\mathrm{d}q + \big(\kappa_f(p)-p\big)\cdot f(p) - \big(\kappa_g(p)-p\big)\cdot g(p) \right| \mathrm{d}p$$

where

- $\overline{\Pi}(f,g)$: all **flow coupling** for distributions f,g

$$\overline{\Pi}(f,g) = \left\{ \pi \in \Delta([0,1]\times[0,1]) : f(p) - g(p) - \int_0^1 \pi(p,q)\mathrm{d}q + \int_0^1 \pi(q,p)dq = 0 \right\}$$

- $\kappa_f(p) = \mathbb{E}_f[Y \mid p]$: true probability underlying prediction p

Theorem IV [Feng, Qian & Tang, EC'26]

Given two **miscalibrated** distribution f,g with [0,1] support and identical means,

$$\text{REMD}^{\text{MisC}}[f,g] = 2\cdot \max_{t\in[0,1]} \left(S_f(t) + \int_0^t \big(p-\kappa_f(p)\big)\cdot f(p)dp \right) - \left(S_g(t) + \int_0^t \big(p-\kappa_g(p)\big)\cdot f(p)\mathrm{d}p \right)$$

Experiment: Infor Gap for LLM Predictions

Experiment Setup

Event predictions:

- Binary outcome $y \in \{0,1\}$
- Each LLM outputs probability $p \in [0,1]$

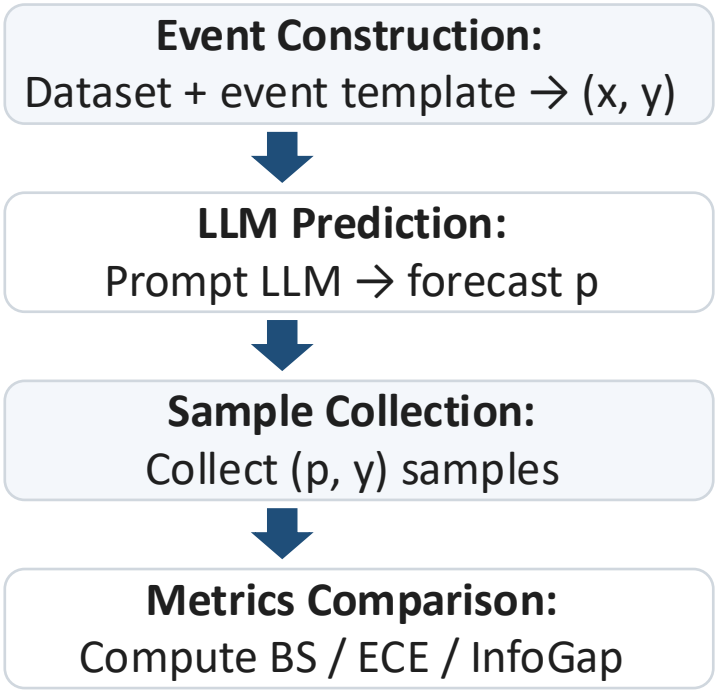
Datasets:

- Weather rain (Kaggle)
- Bitcoin price (CoinMarketCap)

LLM predictors:

- GPT-4o mini &
- DeepSeek-V3.2
- Gemini 2.0 Flash-Lite
- Qwen-Plus

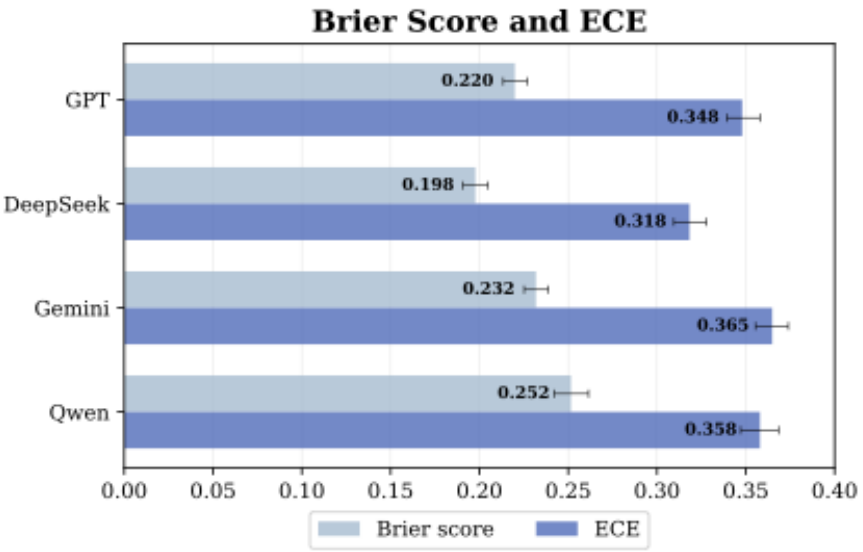
Evaluation pipeline:



Experimental Insight

Infor Gap gives complementary prospective when standard metrics are inconclusive

A representative case study



Highlight (Gemini vs Qwen):

- Brier Score:

0.232 (Gemini) < 0.252 (Qwen)

- ECE: (metrics conflict)

0.365 (Gemini) > 0.358 (Qwen)

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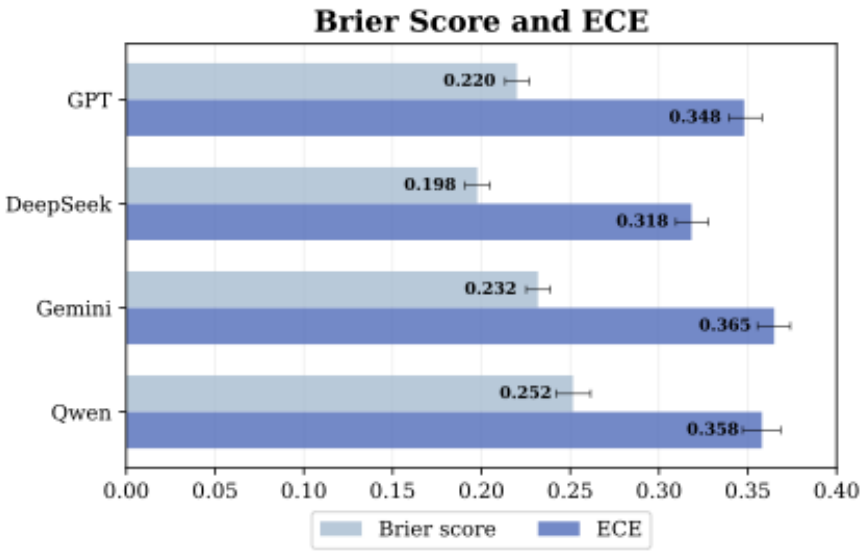

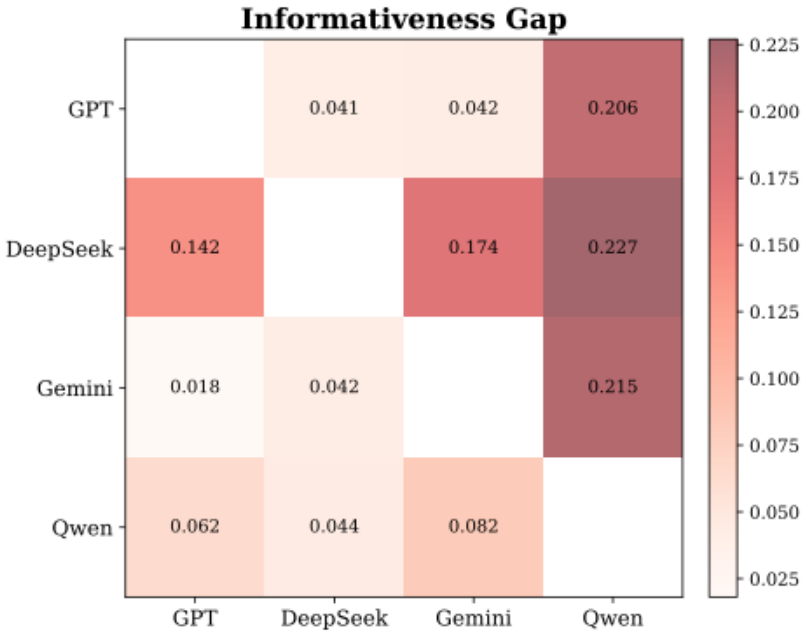

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- Brier Score:

0.232 (Gemini) < 0.252 (Qwen)

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Infor Gap clarifies direction:

- InfoGap[Gemini, Qwen]=0.215
- InfoGap[Qwen, Gemini]=0.082

Applications

- InfoGap to compare different predictors
- Calibrated signaling in digital auctions

Digital Auctions for Ad Impression

2nd –price auction

bid b_i

allocation & payment

Autobidder 1

Autobidder n

ROS/Budget constraint

advertisers

A Simple yet Effective Bidding Strategy

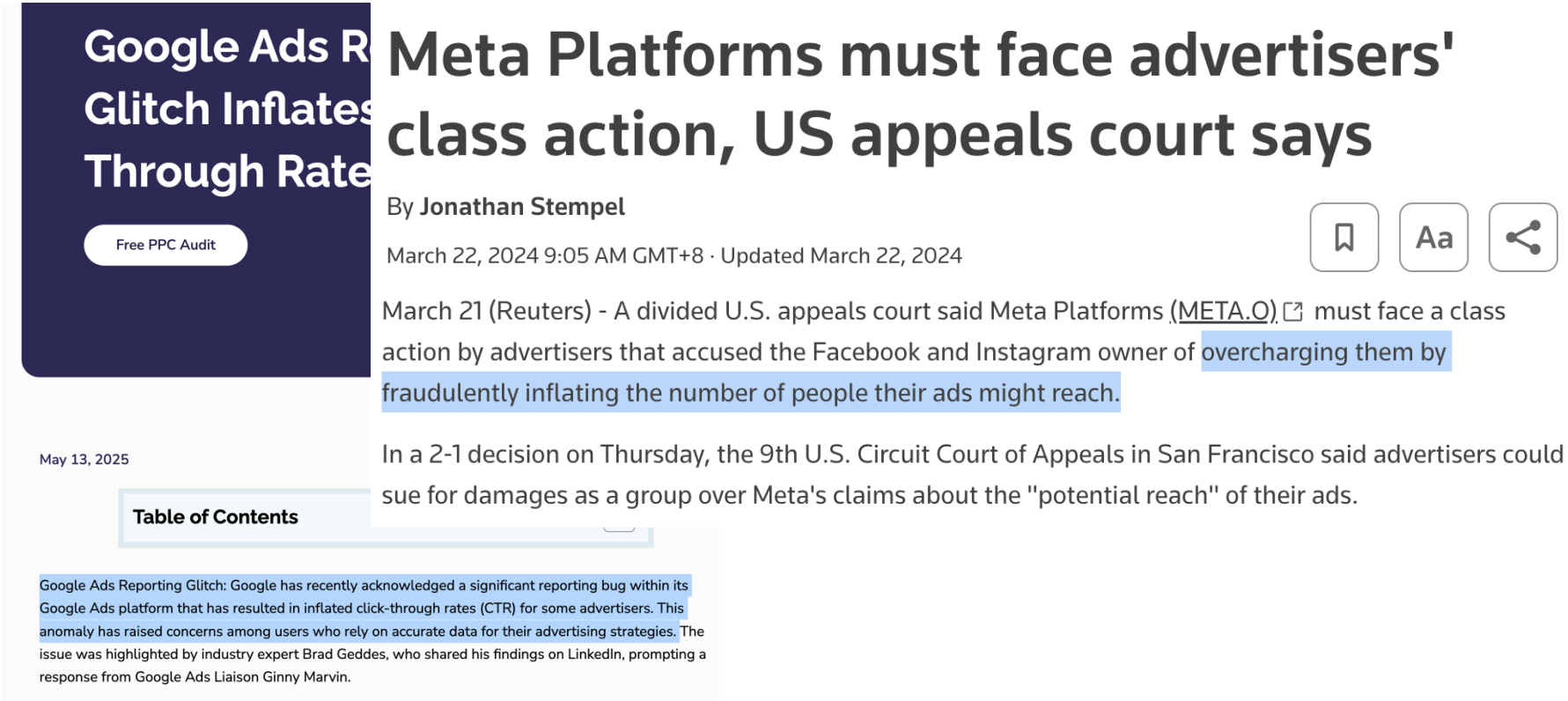
Uniform bidding strategy $b_i \propto c_i \cdot v_i$

- Optimality in many (truthful) auction format [BBW MS'15, ABM WINE'19, DMMZZ WWW'23]

- Performs robustly well v.s optimal non-uniform bidding in many ad auctions [FPMS EC'07, BG MS'19, BFMW EC'14, DLMZ WWW'20, DMMZ WWW'21]

- **Challenge One**: Value v_i typically depends on CTR, and it's unknown to autobidder
  - Solution: Platform needs to inform CTRs to autobidders

How to Credibly Inform CTRs?



Google Ads R
Glitch Inflates
Through Rate

Free PPC Audit

May 13, 2025

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Google Ads Reporting Glitch: Google has recently acknowledged a significant reporting bug within its Google Ads platform that has resulted in inflated click-through rates (CTR) for some advertisers. This anomaly has raised concerns among users who rely on accurate data for their advertising strategies. The issue was highlighted by industry expert Brad Geddes, who shared his findings on LinkedIn, prompting a response from Google Ads Liaison Ginny Marvin.

Meta Platforms must face advertisers' class action, US appeals court says

By **Jonathan Stempel**

March 22, 2024 9:05 AM GMT+8 · Updated March 22, 2024

Aa

March 21 (Reuters) - A divided U.S. appeals court said Meta Platforms (META.O) must face a class action by advertisers that accused the Facebook and Instagram owner of overcharging them by fraudulently inflating the number of people their ads might reach.

In a 2-1 decision on Thursday, the 9th U.S. Circuit Court of Appeals in San Francisco said advertisers could sue for damages as a group over Meta's claims about the "potential reach" of their ads.

- **Challenge Two**: How to calculate CTRs is typically generated through platform's complex internal machine learning algorithms, which are usually considered as trade secrets.
  - Solution: Calibrated signaling

Calibrated Signaling in 2nd-Price Auction

Seller:

- Single-item 2nd-price auction for a finite of n bidders
- Click outcome $\vec{o} = (o_1, o_2, \cdots, o_n) \sim \lambda \in \Delta(\{0,1\}^n)$ **(product dist.)**
  - Seller knows λ, designs a **calibrated signaling scheme** $\boldsymbol{\pi}$:
    - Given outcomes $\vec{o} \sim \lambda$, sends signals $\vec{s} = (s_1, s_2, \cdots, s_n) \sim \pi(\cdot \mid \vec{o})$
  - Each bidder **privately** receives a signal s_i

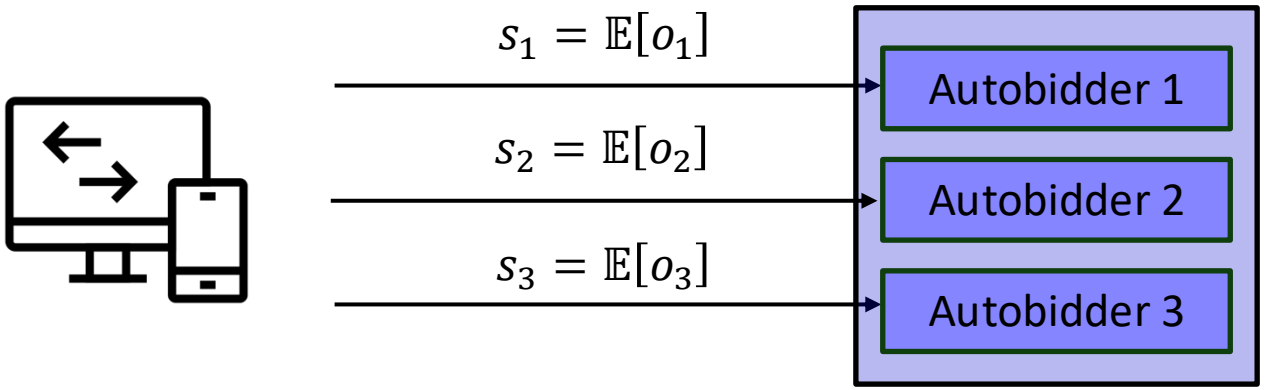
> **Calibrated signaling scheme**: A signaling scheme is calibrated iff: for each bidder i and every possible signal s_i: $\mathbb{E}(o_i \mid s_i) = s_i$,

Autobidders:

- Prior-free: neither know λ nor the signaling details π
- Knowing π is calibrated, simply bid s_i
  - Normalize all $c_i \equiv 1$

Examples of Calibrated Signaling

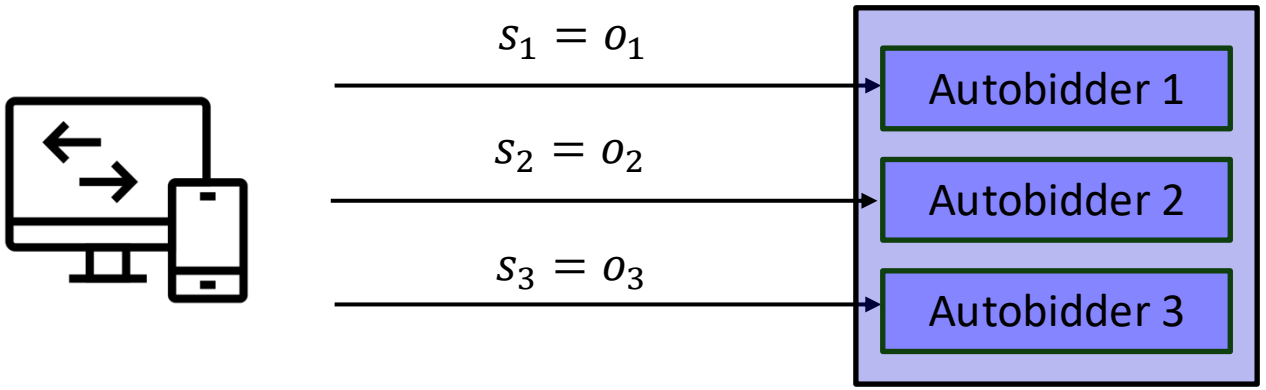
- Let $n = 3$, $\text{Prob}[o_i = 1] = 0.5$
  - No information signaling: $s_i = \mathbb{E}[o_i]$ for all i



$\text{Rev}(\text{No information signaling}) = \mathbb{E}[o_i] = 0.5$

Examples of Calibrated Signaling

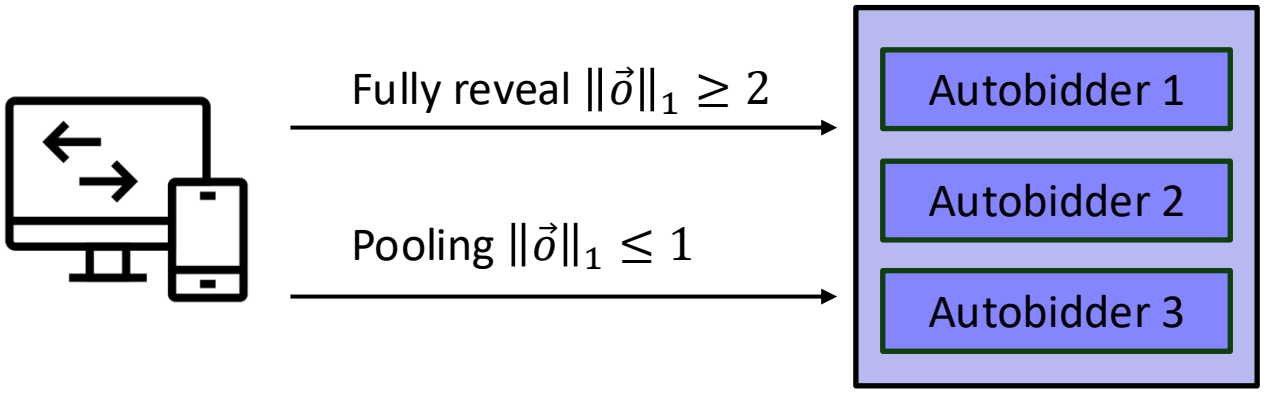
- Let $n = 3, \text{Prob}[o_i = 1] = 0.5$
  - Fully information signaling: $s_i = o_i$ for all i



$$\text{Rev}(\text{Fully information signaling}) = \text{Prob}(\|\vec{o}\|_1 \geq 2) = 0.5$$

Examples of Calibrated Signaling

- Let $n = 3$, $\text{Prob}[o_i = 1] = 0.5$
  - For $\|\vec{o}\|_1 \geq 2$: $\pi(\vec{s} \mid \vec{o}) = 1$ where $\vec{s} = \vec{o}$
  - For $\|\vec{o}\|_1 \leq 1$: $\pi(\vec{s} \mid \vec{o}) = 1$ where $\vec{s} = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$



$$\text{Rev}(\text{Fully + Pooling}) = \text{Prob}(\|\vec{o}\|_1 \geq 2) \cdot 1 + \text{Prob}(\|\vec{o}\|_1 \leq 1) \cdot \frac{1}{4}$$
$$= 0.5 \cdot 1 + 0.5 \cdot \frac{1}{4} > 0.5$$

$$\text{Rev}(\text{Optimal calibrated signaling}) = 0.729$$

Seller's Problem

Revenue Maximizing:

$$\pi^\star = \arg \sup_{\text{calibrated } \pi} \mathbb{E}_{\vec{o}\sim\lambda, \vec{s}\sim\pi(\cdot|\vec{o})}[\text{secmax}(\vec{s})].$$

- secmax($\vec{s}$): second-highest value
- calibration constraint:

$$s = \frac{\sum_{\vec{o}:o_i=1} \lambda(\vec{o}) \int_{s_{-i}} \pi\left((s, s_{-i}) \mid \vec{o}\right) \mathrm{d}s_{-i}}{\sum_{\vec{o}\in\{0,1\}^n} \lambda(\vec{o}) \int_{s_{-i}} \pi\left((s, s_{-i}) \mid \vec{o}\right) \mathrm{d}s_{-i}}, \quad i \in [n], s \in [0,1].$$

- Infinite-dimensional Linear program
- Private information design with n receivers
  - Every $\vec{o}$ is a state
  - Exponential number of states

Related Work

Autobidding: [Survey by Aggarwal et al. SIGecom Exchanges'24] …

Signaling in Auctions:

- In (generalized) 2nd -price auction [Bro Miltersen, Sheffet. EC'12], [Emek et al. TEAC'14], [Badanidiyuru, Bhawalkar, Xu. SODA'18], [Bergemann et al. AER Insights'22], [Bergemann, Duetting, Paes Leme, Zuo. WWW'22], [Chen et al. ICALP'24] …
  - [Bergemann et al. AER Insights'22] considers independent signaling, our work extends to general signaling.

- Joint design of auction and signaling [Bergemann, Pesendorfer. JET'07], [Cai, Li, Wu. EC'24] …

Private information design: [Dughmi and Xu, EC'17], [Arieli, Babichenko. ITCS'19 & JET'22] …

Feasible joint posterior belief: [Morris, 2020],[Brooks et al., ECMA'22], [Arieli et al. EC'20 & JPE'21], [Arieli, Babichenko. EC'22], [Arieli, Babichenko, Sandomirskiy. EC'22], [He, Sandomirskiy, Tamuz. JPE'25], [Yang and Yang. EC'25]

- As noted in [Arieli et al. EC'20 & JPE'21] , characterizing extreme points of feasible joint posterior belief still remains an open question.

Structural Characterization of $\pi^\star$

Theorem [Du, Tang, Wang & Zhang, arXiv'25] The seller-optimal calibrated signaling $\pi^\star$ satisfies that:

1. **[Equal highest- and 2nd-highest bid]** its every signal profile $\vec{s} \in \mathrm{supp}(\pi^\star)$ has the same highest and second-highest bid
2. **[Four signals suffice]** the induced 2nd-highest bid dist. satisfies:

$$\mathrm{Prob}_{\vec{s}\sim\pi^\star(\cdot|\vec{o})}(\mathrm{secmax}(\vec{s})) = \begin{cases} \delta_{(1)}\,, & \|\vec{o}\|_1 \geq 2; \\ \delta_{(t_1^*)}, & \|\vec{o}\|_1 = 1; \\ \delta_{(t_0^*)}\,, & \|\vec{o}\|_1 = 0. \end{cases}$$

where t_1^*, t_0^* are solved via a **linear system** $\mathrm{poly}(n, \lambda)$

- We also characterize optimal calibrated signaling with additional IR constraint

Proof Ideas for Computing $\pi^\star$

Step 1: Symmetrizing calibrated signaling

Lemma: any calibrated signaling π can be **symmetrized** to be $\bar{\pi}$ satisfying following property without hurting any revenue: For any $\|\vec{o}\|_1 = k$, we have

$$\text{Prob}_{\vec{s}\sim\bar{\pi}(\cdot|\vec{o})}(s_i = s) = f_{k,o_i}(s)$$

$\vec{o} = (1,0,\dots,1)$			
signal	$\vec{o} = (0,0,\dots,1)$		
$\vec{s}$	signals	$\vec{o} = (0,1,\dots,1)$	
$\vec{s}'$	$\vec{s}$	signals	$\pi(\vec{s} \mid \vec{o})$
$\vdots$	$\vec{s}'$	$\vec{s}$	0.2
	$\vdots$	$\vec{s}'$	0.6
		$\vdots$	$\vdots$

…

$$\pi\colon \{0,1\}^n \to \Delta([0,1]^n)$$

For any $\vec{o}$ with $\|\vec{o}\|_1 = k$		
bid	$f_{k,o_i=1}(s)$	$f_{k,o_i=0}(s)$
s	0.3	0.5
s'	0.1	0.2
$\vdots$	$\vdots$	$\vdots$

$$\bar{\pi}\colon [n] \to \Delta([0,1]) \times \Delta([0,1])$$

Proof Ideas for Computing $\pi^\star$

Step 2: Reformulated as a two-stage optimization with **optimal transport**

$$\mathrm{REV}[\pi^\star] = \max_{(f_{k,1}, f_{k,0})_{k\in[n]_0} \in \mathcal{F}} \sum_{k\in[n]_0} \lambda_k \cdot \mathrm{REV}^{\mathrm{CoRR}}[f_{k,1}, f_{k,0}]$$

Solving for optimal marginals

The calibration constraint on $\bar{\pi}$ can be reduced to a calibration constraint on $(f_{k,1}, f_{k,0})_{k\in[n]}$

Solving for optimal correlation

k copies $f_{k,1} \in \Delta([0,1])$

$n-k$ copies $f_{k,0} \in \Delta([0,1])$

$\longrightarrow \phi^{\mathrm{CoRR}} \in \Delta([0,1]^n)$

$$\mathrm{REV}^{\mathrm{CoRR}}[f_{k,1}, f_{k,0}] := \max_{\phi^{\mathrm{Corr}}} \mathbb{E}_{\vec{s}\sim\phi^{\mathrm{CoRR}}}[\mathrm{secmax}(\vec{s})]$$

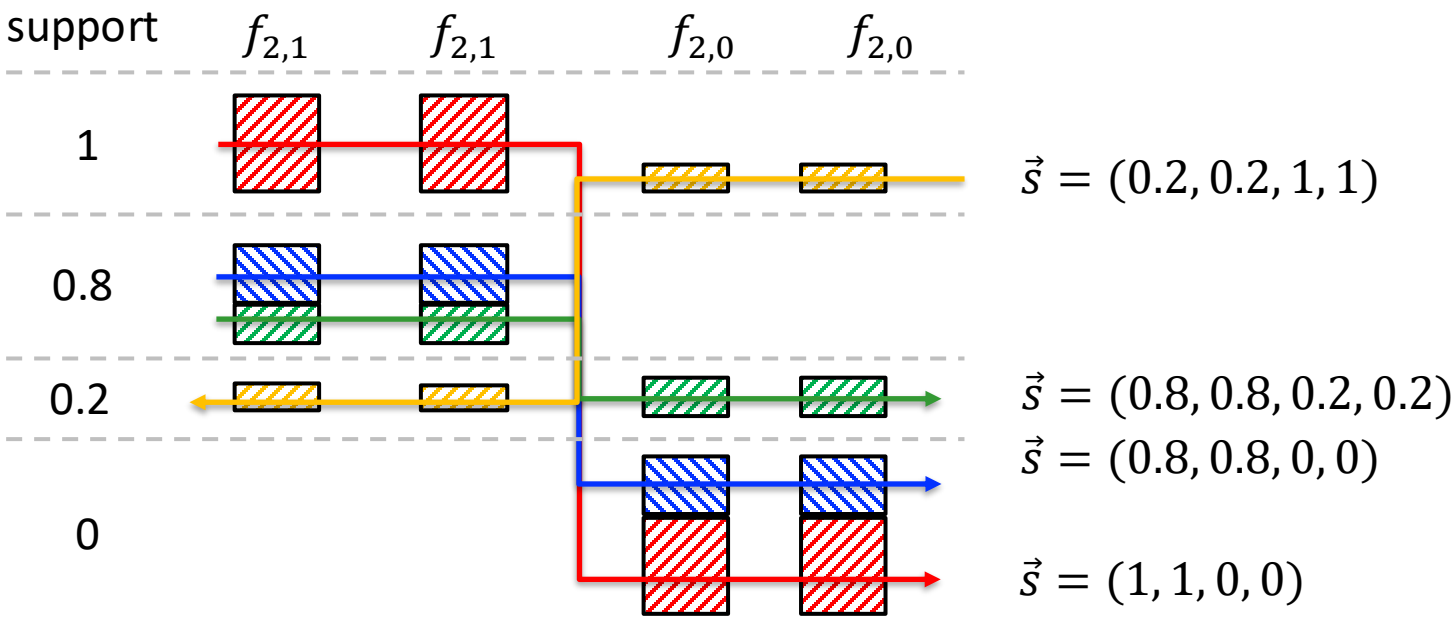
Proof Ideas for Computing $\pi^\star$

Step 3: Solving the optimal correlation:

Lemma: Fix any $k \in [n]$, $\exists$ a "greedy algo." that finds optimal correlation $\phi^{\mathrm{CoRR}} \in \Delta([0,1]^n)$ s.t. it maximizes

$$\mathrm{REV}^{\mathrm{CoRR}}[f_{k,1}, f_{k,0}] \coloneqq \max_{\phi^{\mathrm{CoRR}}} \mathbb{E}_{\vec{s}\sim\phi^{\mathrm{CoRR}}}[\mathrm{secmax}(\vec{s})]$$

$n = 4$
$k = 2$



Proof Ideas for Computing $\pi^\star$

Step 3: Solving the optimal correlation:

Lemma: Fix any $k \in [n]$, $\exists$ a "greedy algo." that finds optimal correlation $\phi^{\mathrm{CoRR}} \in \Delta([0,1]^n)$ s.t. it maximizes

$$\mathrm{REV}^{\mathrm{CoRR}}[f_{k,1}, f_{k,0}] \coloneqq \max_{\phi^{\mathrm{CoRR}}} \mathbb{E}_{\vec{s}\sim\phi^{\mathrm{CoRR}}}[\mathrm{secmax}(\vec{s})]$$

Step 4: With optimal correlation, then solve for optimal marginals:

Lemma: Given the optimal correlation plan, the optimal marginals can be solved via a **linear system** with size $\mathrm{poly}(n, \sum_i \mathrm{bit}(\lambda_k))$ where $\lambda_k = \mathrm{Prob}(\|\vec{o}\|_1 = k)$, and $\mathrm{bit}(\cdot)$ denotes the bit complexity.

Summary

- An intrinsic connection between **calibration** and **information design**
  - **Comparison principle**: How to **compare** different predictors?
  - **Design principle**: How to **design** predictors?
- Many interesting questions:
  - More properties of InfoGap for deterministic predictors?
  - More combinatorial structure on the space of predictors?
  - Beyond binary outcome?

Thanks!

Questions?

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Revenue Characterization