Private Private Information in Second-Price Auction

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Joint work with Boyu Liu[2], Zihe Wang[2], and Shuo Zhang[2]

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The Correlated Information Pitfall

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Badanidiyuru, Bhawalkar, and Xu (2018) show that in second-price auctions, correlated bidder-specific signals can be used to extract nearly all surplus.

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A natural question:

Can information independence *alone* sustain bidders' surplus?

Takeaways from this work

- Once bidders' private information is *correlated*, the bidders' surplus can be fully extracted, and their information rents can be eliminated;
- Even when bidders' private information is ex-ante independent, full surplus extraction remains possible,
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- Information independence, implicitly or explicitly, is suggested to be crucial for sustaining bidders' surplus.
- Information independence is not sufficient to sustain bidders' surplus.

Model: Single-item second-price auction

- N bidders, their valuations $(v_1, \cdots, v_N) \sim \lambda$
  - λ is a fixed and exchangeable prior, publicly known
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- Allocation and payment are determined by the 2^{nd}-price auction

Model: Single-item second-price auction

- Fix an information structure π and a profile of strategies σ, bidder i's ex ante surplus:

$$\mathrm{BS}_i(\pi, \sigma) = \mathbb{E}_{v\sim\lambda,\ s\sim\pi(\cdot|v),\ b\sim\sigma(\cdot|s)}\Big[q_i(b)\big(v_i - \mathrm{secmax}(b)\big)\Big]$$

  - $q_i(b) = \Pr(\text{bidder } i \text{ wins})$
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- We use the Bayes Nash equilibrium (BNE) as the solution concept:

$$\sigma \in \mathsf{BNE}(\pi) \quad \Longleftrightarrow \quad \mathrm{BS}_i(\pi, \sigma) \geq \mathrm{BS}_i\big(\pi, (\sigma_i', \sigma_{-i})\big), \quad \forall i,\ \forall \sigma_i'.$$

Private Private Information

- We require bidders' private information to be ex-ante independent.

Private private information: An information structure is private private if, for every $s = (s_1, \ldots, s_N)$, we have

$$\Pr[s] = \prod_{i \in [N]} \Pr[s_i].$$

– The joint law of $s = (s_1, \ldots, s_N)$ is a product measure.

– Concept formally introduced in He, Sandomirskiy, and Tamuz (2026)

Examples

2 bidders with i.i.d prior: $v_1, v_2 \sim \mathrm{Unif}[0,1]$

- Full-information disclosure: $s_1 \equiv v_1,\ s_2 \equiv v_2$.
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- No-information disclosure: $s_1 = s_2 \equiv \varnothing$.
- Conditionally independent disclosure:
  - Define $\omega = \mathbf{1}\{v_1 + v_2 \geq 1\}$.
  - For each i, independently conditional on v,

$$\pi(s_i = \omega \mid v) = 0.75, \qquad \pi(s_i = 1 - \omega \mid v) = 0.25.$$

  - $\pi(s_1, s_2 \mid v_1, v_2) = \pi(s_1 \mid v_1, v_2) \cdot \pi(s_2 \mid v_1, v_2)$.

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  - $\pi(s_1, s_2 \mid v_1, v_2) = \pi(s_1 \mid v_1, v_2) \cdot \pi(s_2 \mid v_1, v_2)$.
  - But $\Pr(s_1, s_2) \neq \Pr(s_1)\Pr(s_2)$:

$$\Pr(s_1 = 1, s_2 = 1) = \frac{5}{16} \neq \Pr(s_1 = 1)\Pr(s_2 = 1) = \frac{1}{4}$$

Related Work

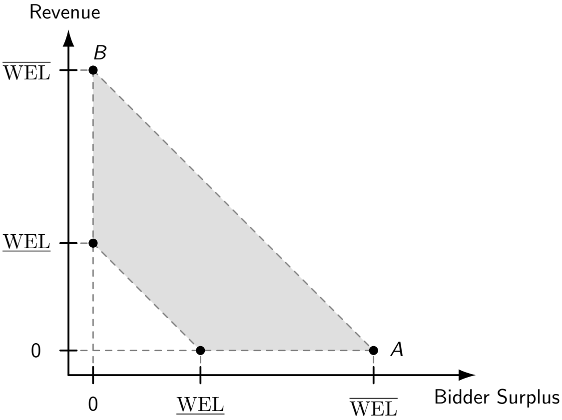
Information design in auctions:

- In (generalized) 2^{nd}-price auctions: [Bro Miltersen and Sheffet, EC'12], [Emek et al., TEAC'14], [Badanidiyuru ea al., SODA'18], [Bergemann et al., AERI'22], [Bergemann et al., WWW'22], [Chen et al., ICALP'24]...
- Other auction formats: [McLean and Postlewaite, RES'04, TE' 15], [Bergemann, Brooks and Morris, ECMA'17, AER'19], [Eden et al., EC'18, FOCS'23], [Chen and Yang, JET'23], [Bo Chen, Jingfeng Lu and Zijia Wang, SSRN'26]...
- Joint design of auctions and signaling: [Bergemann and Pesendorfer, JET'07], [Krähmer, JET'20], [Chen and Yang, JET'23], [Cai, Li and Wu, EC'24], [Bergemann et al., EC'25, JPE'26]...

Equilibria in auctions. [Battigalli and Siniscalchi, GEB'03], [Gomes and Sweeney, EC'09], [Chawla and Hartline, EC'13], [Feldman, Lucier and Noam Nisan, WINE'16], [Deng et al., NIPS'24], [Ahunbay and Bichler, SODA'25], [Baldeschi et al., SODA'26]...

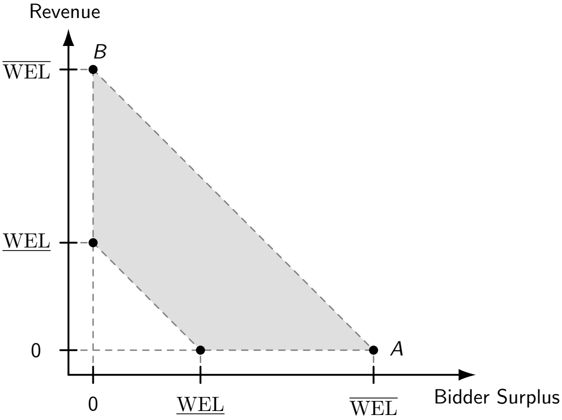
Privacy. [Dwork, ICALP'06], [Dwork et al., TCC'06], [Ichihashi, AER'20], [Liang and Madsen, EC'20], [Eilat, Eliaz and Mu, TE'21], [Schmutte and Yoder, EC'22], [Acemoglu et al., AEJ'22], [He, Sandomirskiy and Tamuz, EC'22, JPE'26], [Schmutte and Yoder, EC'22], [Strack and Yang, ECMA'24]...

Main Results



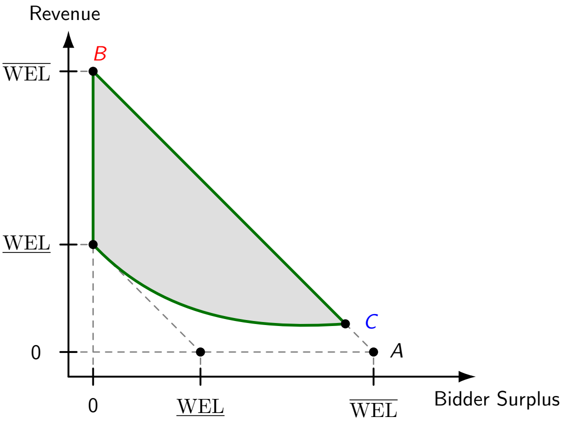
- $\overline{\mathrm{WEL}} = \mathbb{E}_{v\sim\lambda}[\max v]$, $\underline{\mathrm{WEL}} = \mathbb{E}_{v\sim\lambda}[\min v]$.
- $\underline{\mathrm{WEL}} \leq \text{Rev} + \text{Bidder Surplus} \leq \overline{\mathrm{WEL}}$

Main Results: General Information



- $\overline{\mathrm{WEL}} = \mathbb{E}_{v\sim\lambda}[\max v]$, $\quad \underline{\mathrm{WEL}} = \mathbb{E}_{v\sim\lambda}[\min v]$.
- In second-price auctions, achievable outcomes are the shaded trapezoid under **general information structures**.

Known Result



- In **first-price** auctions Bergemann, Brooks, and Morris (2017):
  - **Seller-optimal outcome** (point *B*) can be implemented by a correlated information structure.

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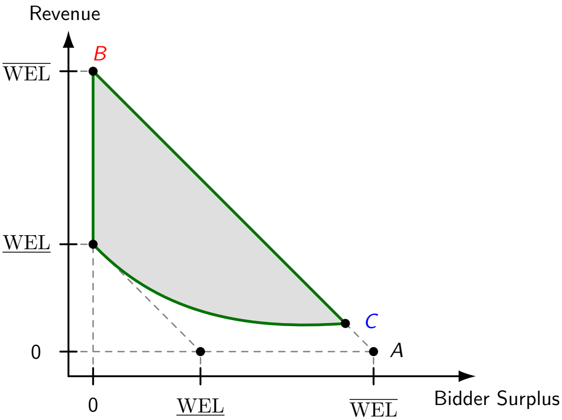
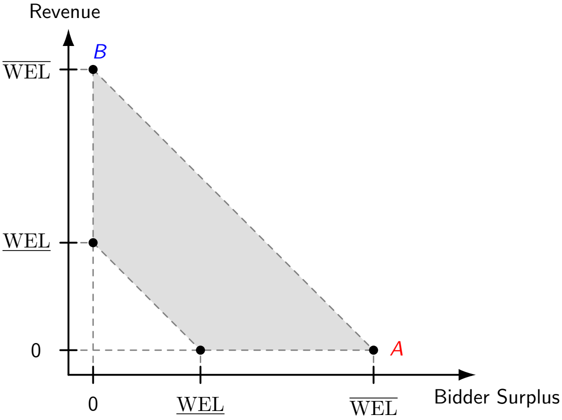


- In **first-price** auctions Bergemann, Brooks, and Morris (2017):
  - **Seller-optimal outcome** (point *B*) can be implemented by a correlated information structure.
  - **Bidder-optimal outcome** (point *C*) can be implemented by a private private information structure.

Main Results: Private Private Information



- Under private private information:
  - The seller can extract the full surplus (point B)

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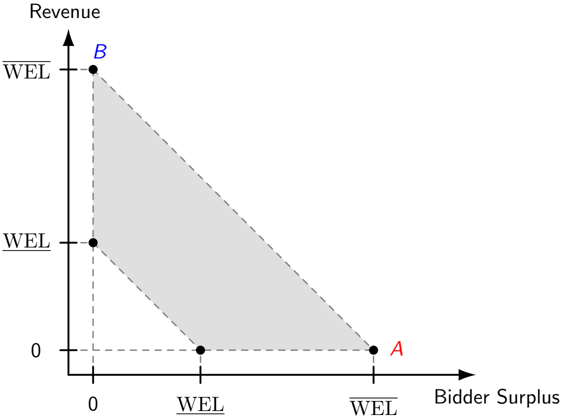
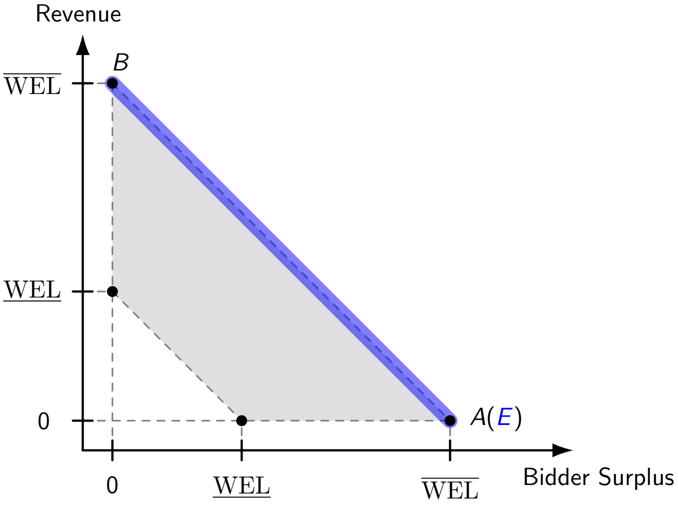


- Under private private information:
  - The seller can extract the full surplus (point B)
  - Bidders cannot generally retain full surplus (point A)

Main Results: Efficient Frontier



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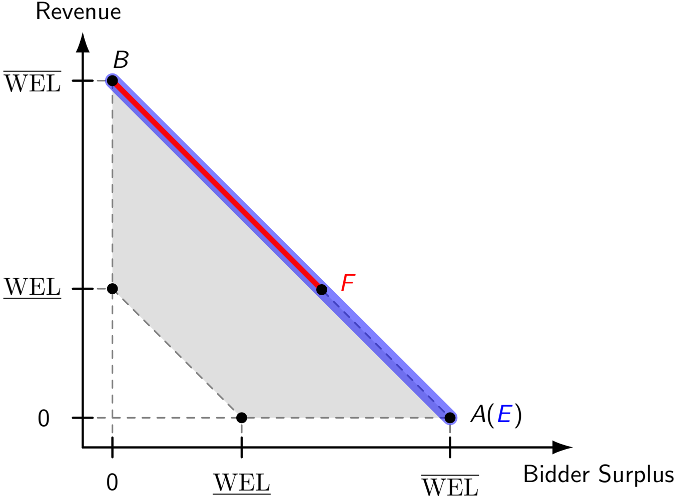

- For common-value setting: Private private efficient frontier is BE
- For IPV setting: Any point in BF can be implemented by a private private information where F is full-information disclosure outcome

Seller's Problem

$$\sup_{\pi:\ \text{private-private}} \mathbb{E}_{\substack{v\sim\lambda,\ s\sim\pi(\cdot|v)\\ b\sim\sigma(\cdot|s)}} \left[\mathrm{secmax}(b)\right].$$

- σ is a BNE under the private private information π
- Private-private constraint:

$$\Pr[s] = \prod_{i\in[N]} \Pr[s_i].$$

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3. Infinite-dimensional design space – *continuous optimization*.

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2. **[Uniform marginals]** *for all $i \in [N]$, the marginal distribution of s_i under π is a uniform distribution*
3. **[Permutation invariant]** *for every permutation ξ and every $(v, s) \in V^N \times S^N$:*

$$\pi(v, s) = \pi(\xi(v), \xi(s))$$

Two Important Conditions

Winner-Dominance. *For any realized signal profile* s *and any bidder* i*, if she receives the highest signal, i.e.,* $s_i = \max(s)$*, then she must hold the highest valuation, i.e.,* $v_i = \max(v)$*.*

- *The item is always assigned to the highest-value bidder*
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No-Rent Winner. *For any realized signal profile s and any bidder i, if $s_i = \max(s)$, her posterior expected value $x_i(s)$ equals the second -highest signal, i.e., $x_i(s) = \mathrm{secmax}(s)$.*

– *Always charge the winner her expected posterior value*

– *Guarantee the* **full surplus extraction**

Two Key Conditions

Proposition. *For any info. structure satisfying Winner-Dominance and No-Rent Winner, bidder directly bids their signal,* $\sigma_i(s_i) = \delta_{s_i}$, *is an* **efficient** *BNE. Under this equilibrium, bidders earn* **zero surplus***.*

- The remaining task is to construct a private-private information structure satisfying both conditions.

Construction Ideas: A Two-Bidder Example

Two independent values $v_1, v_2 \sim \mathrm{Unif}([0,1])$ and a common uniform signal space $\bar{S} = [\underline{s}, \bar{s}]$

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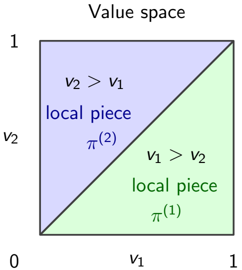

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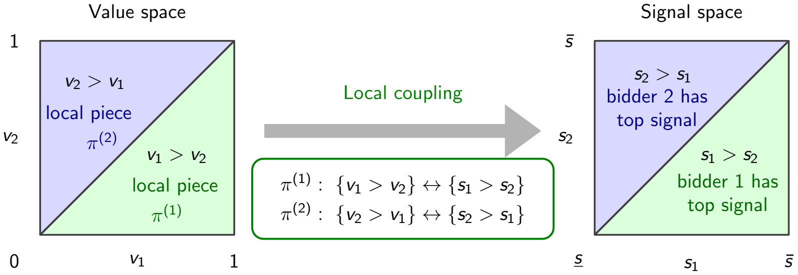

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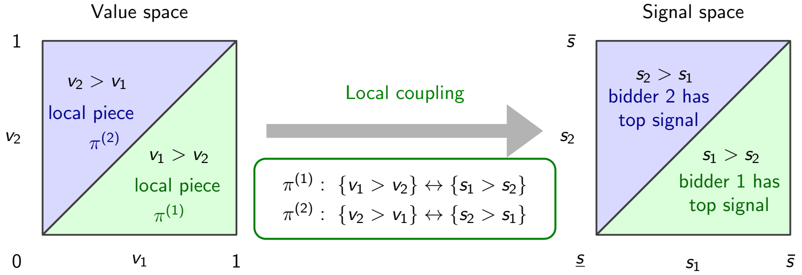

– $\pi^{(i)}$ matches $V^{(i)} = \{v : v_i = \max(v)\}$ and $S^{(i)} = \{s : s_i = \max(s)\}$

– The distribution on $V^{(i)}$ and $S^{(i)}$ is fixed, we denote as $\lambda^{(i)}$ and $\phi^{(i)}$

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  - The No-Rent Winner condition pins down bidder i's posterior mean:

$$x_i(s) \equiv \mathbb{E}_{\pi^{(i)}}[v_i \mid s] = \mathrm{secmax}(s), \qquad \forall s \in S^{(i)}.$$

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  - Linear system for the local coupling $\pi^{(i)}$:

$$\begin{array}{rlcl} \text{marginals:} & \pi_V^{(i)} = \lambda^{(i)}, \quad \pi_S^{(i)} = \phi^{(i)} & \Longrightarrow & \text{Winner-Dominance}, \\ \text{posterior mean:} & \mathbb{E}_{\pi^{(i)}}[v_i \mid s] = \mathrm{secmax}(s), \quad \forall s \in S^{(i)} & \Longrightarrow & \text{No-Rent Winner}. \end{array}$$

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Lemma. *There always exists a common signal space $\bar{S}$ such that the linear system is feasible.*

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Lemma. *The averaged joint coupling $\pi \in \Delta(V^2 \times \bar{S}^2)$ has marginals $\mathrm{Unif}([0,1]^2)$ and $\mathrm{Unif}(\bar{S}^2)$, and satisfies both conditions.*

Construction Ideas: A Two-Bidder Example

Two independent values $v_1, v_2 \sim \mathrm{Unif}[0,1]$ and a common uniform signal space $\bar{S} = [\underline{s}, \bar{s}]$

- **Step 2**: Assemble the Local Couplings.
  - Symmetric mixture:

$$\pi(v,s) = \frac{1}{2}\pi^{(1)}(v,s) + \frac{1}{2}\pi^{(2)}(v,s)$$

Lemma. *The averaged joint coupling $\pi \in \Delta(V^2 \times \bar{S}^2)$ has marginals $\mathrm{Unif}([0,1]^2)$ and $\mathrm{Unif}(\bar{S}^2)$, and satisfies both conditions.*

$$\pi(v_1, v_2, s_1, s_2) = \begin{cases} \dfrac{2}{v_1(1-s_2)} \cdot \mathbf{1}\left\{\dfrac{1}{2} \le s_2 \le s_1 \le 1,\ s_2 \le \dfrac{v_1+1}{2}\right\}, & \text{if } v_1 > v_2, \\ \dfrac{2}{v_2(1-s_1)} \cdot \mathbf{1}\left\{\dfrac{1}{2} \le s_1 \le s_2 \le 1,\ s_1 \le \dfrac{v_2+1}{2}\right\}, & \text{if } v_2 > v_1. \end{cases}$$

Conclusion

Main takeaways

- In second-price auctions, full surplus extraction is possible under private-private information.
- Full bidder surplus cannot generally be achieved under private private information.
- For common-value and IPV environments, we characterize parts of the efficient frontier.

Conclusion

Main takeaways

- In second-price auctions, full surplus extraction is possible under private-private information.
- Full bidder surplus cannot generally be achieved under private private information.
- For common-value and IPV environments, we characterize parts of the efficient frontier.

Future work

- Identify the conditions under which private private information arises endogenously.

Thanks!

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Paper Link